Appendix A

This appendix presents a derivation of Eqs. (3)-(4) of the paper “observation of two-photon speckle”. Inspired by one-photon speckle analysis [1], we approximate the autocorrelation function of the two-photon field in the exit plane of the scatterer by a quasi-homogeneous function. In such a form the autocorrelation function separates into a wide function of the average coordinate and a narrow function of the difference coordinate, i.e.,

$$\Gamma(x_1, x_2; \delta x_1, \delta x_2) \equiv A^{\text{ex}}_x(x_1^+, x_2^+) A^{\text{ex}}_x(x_1^-, x_2^-) \approx R^{\text{ex}}(x_1, x_2) \times f_{\text{ex}}(\delta x_1, \delta x_2)$$  \tag{A.1}

where $x_{1,2}^\pm \equiv x_{1,2} \pm \frac{1}{2} \delta x_{1,2}$, $A^{\text{ex}}_x(x_1, x_2)$ is the two-photon field in the exit plane of the scatterer, and $R^{\text{ex}}(x_1, x_2)$ is the average two-photon intensity in the exit plane. The compact function $f_{\text{ex}}(\delta x_1, \delta x_2)$ is related to the opening angle of the scattered light. We associate this approximation with strongly scattering surfaces with large surface height fluctuations ($\gg \lambda$) and a transverse correlation of the surface roughness that is short as compared to the structure in the average two-photon intensity (strong-scattering regime).

Next, we propagate $\Gamma_{\text{ex}}$ to the far-field plane to obtain $\Gamma_{\text{FF}}$ being the two-photon field correlation function in the far-field plane. Spatial propagation of a two-photon field is described by applying the electric field propagator to each coordinate individually, yielding [2]

$$A_{\text{FF}}(s_1, s_2) = \int dx_1 \int dx_2 h(s_1, x_1) h(s_2, x_2) A_{\text{ex}}(x_1, x_2), \tag{A.2}$$

where $s_{1,2}$ are the transverse positions in the far-field plane and

$$h(s, x) = \frac{k}{2\pi f_d} \exp \left(-ik \frac{s \cdot x}{f_d}\right), \tag{A.3}$$

is the far-field propagator of the electric field, $f_d$ is the focal length of the far-field imaging lens, and $k$ is the radial wavenumber of a down-converted photon in vacuum. By combining Eqs. (A.1)-(A.3) one obtains the two-photon field correlation function in the far-field plane

$$\Gamma_{\text{FF}}(s_1, s_2; \delta s_1, \delta s_2) = \left(\frac{k}{2\pi f_d}\right)^4 \mathfrak{F} \left[\mathfrak{F}^{-1}[\Gamma_{\text{ex}}(x_1, x_2)] \left(k \frac{\delta s_1}{f_d}, k \frac{\delta s_2}{f_d}\right) \times \mathfrak{F}[f_{\text{ex}}(\delta x_1, \delta x_2)] \left(k \frac{s_1}{f_d}, k \frac{s_2}{f_d}\right)\right], \tag{A.4}$$

where $\mathfrak{F}$ denotes spatial Fourier transform. This correlation function has a similar separable form as Eq. (A.1).

Finally, we assume Gaussian statistics of the two-photon field in the far-field of the scatterer. This assumption requires high-dimensional entanglement of the input field; it becomes approximate for small Schmidt numbers (see Fig. 2 in Ref. [3]). The complex Gaussian moment theorem [1] now yields an expression for the desired coincidence correlation function $C$ in terms of $\Gamma_{\text{FF}}$:

$$C(s_1, s_2; \delta s_1, \delta s_2) = \frac{\Gamma_{\text{FF}}(s_1, s_2; \delta s_1, \delta s_2)}{\Gamma_{\text{FF}}(s_1, s_2)^2} = \left[1 + \left|\mu'(\delta s_-, \delta s_+\right)\right], \tag{A.7}$$

where $\delta s_{1,2} \equiv s_{1,2} \pm \frac{1}{2} \delta s_{1,2}$, and where we once more used strong scattering in the second line.

By inserting Eq. (A.4) for $\Gamma_{\text{FF}}$ we find

$$C(s_1, s_2; \delta s_1, \delta s_2) = \frac{\mathfrak{F} \left[R^{\text{ex}}(x_-, x_+)^* \right] \left(k \frac{\delta s_1}{f_d}, k \frac{\delta s_+}{f_d}\right)}{\mathfrak{F} \left[R^{\text{ex}}(x_-, x_+)^* \right] \left(0, 0\right)}, \tag{A.8}$$

and $\delta s_{\pm} \equiv \frac{1}{\sqrt{2}}(\delta s_1 \pm \delta s_2)$ and where the shape function

$$\mu'(\delta s_-, \delta s_+) = \frac{\mathfrak{F} \left[R^{\text{ex}}(x_-, x_+)^* \right] \left(k \frac{\delta s_1}{f_d}, k \frac{\delta s_+}{f_d}\right)}{\mathfrak{F} \left[R^{\text{ex}}(x_-, x_+)^* \right] \left(0, 0\right)}, \tag{A.8}$$

is Fourier related to the average two-photon intensity in the exit plane $R^{\text{ex}}(x_-, x_+)$, expressed in a rotated basis where $x_{\pm} \equiv \frac{1}{\sqrt{2}}(x_1 \pm x_2)$. Equations (A.7)-(A.8) become Eqs. (3)-(4) in the main text after a simple conversion to angles.

In the above derivation of the correlation functions, we neglected the effect of the symmetry constraint of the two-photon field $A_{\text{ex}}(x_1, x_2) = A_{\text{ex}}(x_2, x_1)$. This symmetry results in average photon bunching close to the diagonal $s_1 \approx s_2$. The approximation is allowed since the number of spatial modes is sufficiently large for the considered case of strong scattering and high-dimensional entanglement. In this regime, the vast majority of the two-photon speckle spots is located far away from the diagonal, where $s_1 \neq s_2$ and the effect of photon bunching is not relevant.
